Functions

Definition: A *set* is a collection of objects.

Notation: We write $a \in A$ to denote that the object a is an element of the set A.

Definition: Let A and B be be sets. A *function* with domain A and codomain B is a rule that, to each element of A, assigns an unambiguous element of B.

Notation: We will write $f : A \rightarrow B$ to mean that f is a function with domain A and codomain B.

How can you tell if something is a function? The definition above says there are two "tests" a function must pass:

1. Each element in A must have something assigned to it.

2. Each element in A must have only one thing assigned to it, (so the object that gets assigned is unambiguous).

If f passes these tests, then f is a function.

Exercises:

1. Let A denote the set of all people living in Chicago. Let B denote the set of all telephone numbers. Let f assign, to each person in A, their telephone number. Is this a function? Explain in complete sentences.

2. Let A denote the set of all people who live in Chicago and own exactly one phone. Let B denote the set of all telephone numbers. Let f assign, to each person in A, their telephone number. Is f a function? Explain in complete sentences.

3. Let A denote the set of all people who were alive in 2001. Let B denote the set of numbers. Let f assign, to each person in A, their weight during the year 2001. Is f a function? Explain in complete sentences.

4. Let A denote the set of all people (who are living or dead). Let B also denote the set of all people (who are living or dead). Let f assign, to each person in A, their biological mother. Is f a function? Explain in complete sentences.

5. You go for a walk from your house at 12:00 noon and return at 2:00. Let A denote the set of numbers between 0 and 2 (inclusive) and let B be the set of all numbers. The numbers in A will represent times after 12:00 in hours. Let f assign, to each number in A, the distance you were from home at that time. Is f a function? Explain in complete sentences.

6. You go for a walk from your house at 12:00 noon and return at 2:00. The farthest you walked from home was 3 miles. Let A denote the set of numbers between 0 and 3 (inclusive) and let B be the set of all numbers. The numbers in A will represent the distances you were from home during your walk. Let f assign, to each distance in A, the time when you were that distance from home. Is f a function? Explain in complete sentences.

7. Let A denote the set of all numbers. Let B denote the same set as A. Let f be the function that, to each number in A assigns the square of the number. Is f a function? Explain in complete sentences.